J. Fluid Mech. (1968), *vol.* 34, *part* 4, *pp.* 759–768
Printed in Great Britain

Long standing waves in a curved canal

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(Received 8 February 1968)

The dynamics of long water waves are considered in a curved geometry representing a canal bend. The presence of the bend is found to produce a spectrum of transverse oscillations in the canal. The associated dominant amplitudes are evaluated for both tidal periods and higher frequencies representative of tsunamis. It is found that low-frequency waves do not lead to significant transverse amplitudes. For tsunamis, the presence of the bend may result in considerable changes in the local wave amplitude.

1. Introduction

Recent theoretical investigations into estuarial tidal flows have included contributions by Hunt (1964) and Johns (1966, 1967). Although each of these authors accounted for topographical features to a varying degree, the direct presence of a bend in a river estuary was neglected. In the methods used therein, it was implicitly assumed that all dynamical conditions were solely dependent upon a curvilinear co-ordinate along the middle of the river. It is natural, therefore, to inquire to what extent the resulting formulae are likely to be modified by the inclusion of a curved geometry to represent a bend. We may ask, for example, whether it will produce a dynamical effect that extends (significantly) upstream and downstream of the bend. Secondly, will the bend induce a perceptible transverse surface slope in the estuary comparable with that known to be associated with the Coriolis force? (Abbot 1960.) Principally, it is with a view to answering these questions that the present paper is concerned. At the same time, however, the theory of the subsequent paragraphs is applicable to the propagation of tsunamis, or other long-period waves, into a curved canal, and numerical computations are undertaken in both cases.

In order to assess the gross effect of a curved geometry on long waves, we consider here the dynamics of standing waves in a curved canal. The methods used depend upon the application of suitable co-ordinate systems in different sections of the canal. The formulae are then evaluated numerically in the following cases: (i) for a semi-diurnal tidal wave in a curved canal of estuarial dimensions; (ii) for a higher-frequency wave with a period representative of a tsunami.

It is found that the curved geometry induces within the canal a spectrum of transverse oscillations. The numerical value of the amplitudes of these oscilla-

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tions is found to be closely related to the frequency of the primary wave. In case (i) the low tidal frequency effectively inhibits the generation of significant transverse oscillations, and the dynamics may accordingly be evaluated by use of a rectilinear geometry. In case (ii), for a wave with a period $\sim$ 3–4 min, the local amplitude of the dominant transverse oscillation can amount to as much as 20 % of that of the primary wave. The presence of a bend may therefore lead to a significant increase in the destructive capacity of a tsunami.

2. Formulation

The bend in the canal is represented by the region between the arcs of two concentric circles having radii a and b with $a < b$. The angle subtended by each of these arcs at the centre of its associated circle is θ_1 and the angular position of a general point in the curved region is specified by θ, which is such that $\theta = \frac{1}{2}\pi$ and $\theta = \theta_1 + \frac{1}{2}\pi$ at the ends of the circular section. The continuation of the canal for $\theta < \frac{1}{2}\pi$ and $\theta > \theta_1 + \frac{1}{2}\pi$ is represented by straight parallel-sided channels of width $b-a$, the lateral boundaries of which are tangential to the circular arcs at $\theta = \frac{1}{2}\pi$ and $\theta = \theta_1 + \frac{1}{2}\pi$. In the subsequent paragraphs, the method of solution of the equations governing frictionless shallow-water oscillations in the canal will depend upon the use of a different horizontal axis system in each region. These will be amenable to the local geometry. Their orientation, with respect to the given configuration, is shown in figure 1.

The z-axis is measured vertically upwards from the undisturbed level of the water in the canal. Supposing that the water is of constant mean depth h, and that the instantaneous position of the free surface is denoted by $z = \zeta$, then, in the absence of the Coriolis acceleration, the linearized shallow-water equations lead to

$$gh\Delta\zeta = \partial^2\zeta/\partial t^2, \tag{2.1}$$

where Δ is the two-dimensional horizontal Laplacian operator. Specifying an oscillation having an angular frequency σ, a solution of (2.1) is represented by $Z\cos\sigma t$ and so Z must satisfy

$$\Delta Z + \lambda^2 Z = 0, \tag{2.2}$$

where

$$\lambda^2 = \sigma^2/gh.$$

In the curved region (1), we use cylindrical co-ordinates (r, θ, z) in terms of which the solution Z_1 must satisfy

$$\frac{\partial^2 Z_1}{\partial r^2} + \frac{1}{r}\frac{\partial Z_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 Z_1}{\partial \theta^2} + \lambda^2 Z_1 = 0 \quad (a \leqslant r \leqslant b; \quad \tfrac{1}{2}\pi \leqslant \theta \leqslant \theta_1 + \tfrac{1}{2}\pi). \tag{2.3}$$

The accompanying boundary conditions must be compatible with a zero radial velocity on the lateral walls:

$$\partial Z_1/\partial r = 0 \quad \text{at} \quad r = a, b. \tag{2.4}$$

Within the rectilinear region (2), the use of horizontal Cartesian co-ordinates (x, y) in (2.2) leads to

$$\frac{\partial^2 Z_2}{\partial x^2} + \frac{\partial^2 Z_2}{\partial y^2} + \lambda^2 Z_2 = 0 \quad (a \leqslant y \leqslant b; \quad x \geqslant 0), \tag{2.5}$$

whilst the appropriate conditions to be applied at the lateral boundaries yield the requirements

$$\partial Z_2/\partial y = 0 \quad \text{at} \quad y = a, b. \tag{2.6}$$

Likewise, in the region (3), we have

$$\frac{\partial^2 Z_3}{\partial X^2}+\frac{\partial^2 Z_3}{\partial Y^2}+\lambda^2 Z_3 = 0 \quad (a \leqslant Y \leqslant b; \quad X \leqslant 0), \tag{2.7}$$

with

$$\partial Z_3/\partial Y = 0 \quad \text{at} \quad Y = a, b. \tag{2.8}$$

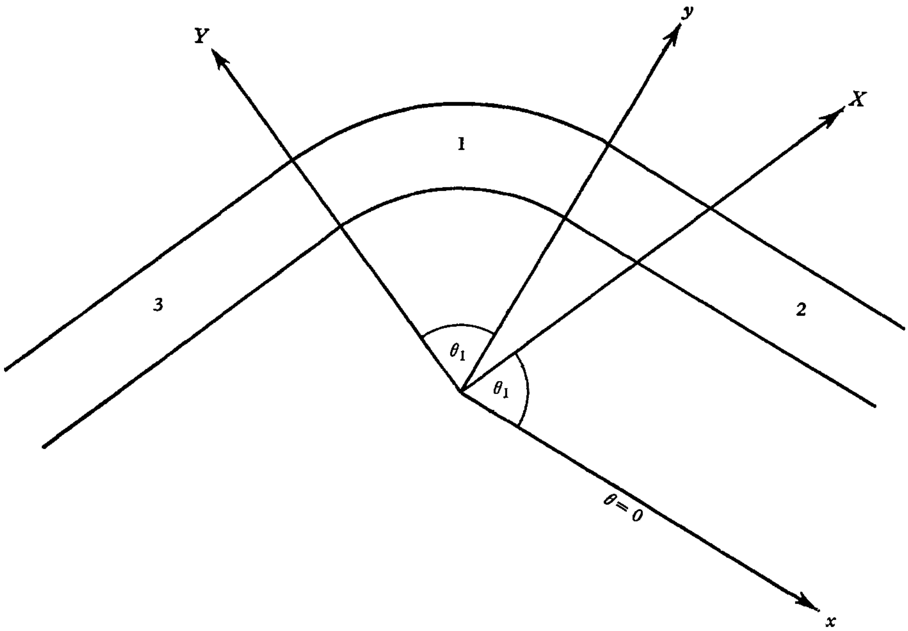

FIGURE 1. Diagrammatic sketch of co-ordinate system.

The appropriate solutions of the foregoing system of equations must be compatible with a continuity of conditions at positions of common validity. With regard to both elevation and longitudinal current, this is readily found to yield

$$\left.\begin{aligned} Z_1 &= Z_2 \\ \frac{1}{r}\frac{\partial Z_1}{\partial \theta} &= -\frac{\partial Z_2}{\partial x} \end{aligned}\right\} \quad \text{at } \theta = \tfrac{1}{2}\pi, \quad a \leqslant r \leqslant b, \tag{2.9}$$

and

$$\left.\begin{aligned} Z_1 &= Z_3 \\ \frac{1}{r}\frac{\partial Z_1}{\partial \theta} &= -\frac{\partial Z_3}{\partial X} \end{aligned}\right\} \quad \text{at } \theta = \theta_1+\tfrac{1}{2}\pi, \quad a \leqslant r \leqslant b. \tag{2.10}$$

It is not difficult to show that these conditions automatically imply the continuity of transverse current.

3. Solution of equations

In the present work it is stipulated that the solutions of (2.5) and (2.7) shall be representative of standing waves (uniform across the canal) at a great distance upstream and downstream from the curved region. For sufficiently large values of x and $-X$ we must therefore have

$$Z_2 \sim A\cos\lambda x + B\sin\lambda x \tag{3.1}$$

and

$$Z_3 \sim G\cos\lambda X + H\sin\lambda X. \tag{3.2}$$

The effect of the bend will be to generate transverse oscillations in regions (2) and (3) that must attenuate with distance in order to be compatible with (3.1) and (3.2). With this in mind, it is readily found that a suitable function fulfilling (2.5) and (2.6) is of the form

$$\exp(-p_n x)\cos\left\{\frac{n\pi(y-a)}{b-a}\right\} \quad (n = 1, 2, \ldots), \tag{3.3}$$

where

$$p_n^2 = \frac{(n\pi)^2}{(b-a)^2} - \lambda^2. \tag{3.4}$$

In order to secure a real value of p_n, we must also have that

$$\lambda(b-a) < \pi, \tag{3.5}$$

which, for long-period oscillations in canals of realistic dimensions, is always satisfied. By virtue of the linearity of (2.5) we may therefore write an appropriate solution in the form

$$Z_2 = A\cos\lambda x + B\sin\lambda x + \sum_{n=1}^{\infty} C_n \exp(-p_n x)\cos\left\{\frac{n\pi(y-a)}{b-a}\right\}. \tag{3.6}$$

The constants A and B are prescribed by the amplitude and spatial phase of the oscillation, whilst C_n is to be determined by application of the juncture conditions (2.9) and (2.10). Likewise, in region (3), we obtain

$$Z_3 = G\cos\lambda X + H\sin\lambda X + \sum_{n=1}^{\infty} D_n \exp(p_n X)\cos\left\{\frac{n\pi(Y-a)}{b-a}\right\}, \tag{3.7}$$

where G, H and D_n are all to be found in terms of A and B.

Within the region (1), we propose a basic solution of (2.3) in the form

$$Z_1 = R(r)\,\Theta(\theta), \tag{3.8}$$

R and Θ being respectively functions of r and θ only. It is found that we may fulfil (2.3) by choosing R and Θ to satisfy the equations

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left(\lambda^2 - \frac{K^2}{r^2}\right)R = 0 \tag{3.9}$$

and

$$\frac{d^2\Theta}{d\theta^2} + K^2\Theta = 0, \tag{3.10}$$

where the separation constant K may be either real or imaginary. A real value of K will lead to a solution which is a linear combination of $\cos K\theta$ and $\sin K\theta$, this clearly being the part required to communicate the basic standing wave (3.1) into region (3). An imaginary value of K will lead to that part of the solution responsible for the transverse oscillations in (3.6) and (3.7). As far as the real value of K is concerned, we may reasonably expect that $K = O(\lambda a)$ in order to correspond approximately to the situation in regions (2) and (3). More precisely, however, we may solve (3.9) for real and non-integral values of K and obtain a solution for R in the form of a linear combination of Bessel functions:

$$R = PJ_K(\lambda r) + QJ_{-K}(\lambda r). \tag{3.11}$$

In order that (3.11) shall lead to the fulfilment of (2.4), we may choose P and Q so that

$$R = \phi_K(\lambda r) = J'_K(\lambda a) J_{-K}(\lambda r) - J'_{-K}(\lambda a) J_K(\lambda r), \tag{3.12}$$

where

$$\phi'_K(\lambda b) = 0, \tag{3.13}$$

and the primes denote a differentiation with respect to the arguments of the various functions. The relation (3.13) is therefore a characteristic equation from which the exact value of K is to be determined.

For $K^2 < 0$ we write

$$K = i\mu, \tag{3.14}$$

and, by a similar analysis, find a solution for R in which

$$R = \phi_{i\mu}(\lambda r), \tag{3.15}$$

where

$$\phi'_{i\mu}(\lambda b) = 0. \tag{3.16}$$

As will be shown later, there exists an infinite sequence of real values of μ satisfying (3.16). By a superposition of basic solutions, we may therefore write the full solution for Z_1 in the form

$$Z_1 = \phi_K(\lambda r)(\alpha \cos K\theta + \beta \sin K\theta) + \sum_{s=1}^{\infty} \phi_{i\mu_s}(\lambda r)\{\gamma_s \exp(\mu_s \theta) + \delta_s \exp(-\mu_s \theta)\}. \tag{3.17}$$

The arbitrary constants in (3.17) will be chosen so as to communicate the complete oscillations between the rectilinear canals (2) and (3).

In general, the solution of the transcendental equation (3.13) is obtainable only by a numerical process. In the present work, however, we use an expansion technique in order to construct a suitable approximate procedure. We may write

$$\phi'_K(\lambda b) = \phi'_K(\lambda a + \epsilon), \tag{3.18}$$

where

$$\epsilon = \lambda(b-a), \tag{3.19}$$

and so, by a formal application of Taylor's theorem,

$$\phi'_K(\lambda b) = \sum_{n=0}^{\infty} \frac{\epsilon^n}{n!} \phi_K^{(n+1)}(\lambda a). \tag{3.20}$$

Upon use of (3.12) in (3.20),

$$\phi'_K(\lambda b) = \sum_{n=0}^{\infty} \frac{\epsilon^n}{n!} \{J'_K(\omega) J_{-K}^{(n+1)}(\omega) - J'_{-K}(\omega) J_K^{(n+1)}(\omega)\}, \tag{3.21}$$

where

$$\omega = \lambda a.$$

The coefficients of the successive powers of ϵ in (3.21) are readily expressible in terms of elementary functions (Watson 1958), and we obtain

$$\phi'_K(\lambda b) = -\frac{\epsilon \sin K\pi}{\pi\omega}\left\{2\left(1-\frac{K^2}{\omega^2}\right)+\frac{\epsilon}{\omega}\left(\frac{3K^2}{\omega^2}-1\right) -\frac{\epsilon^2}{3}\left(\frac{K^4+11K^2}{\omega^4}-\frac{2K^2+3}{\omega^2}+1\right)\right\}+O(\epsilon^4). \quad (3.22)$$

If ϵ be in some sense small, the use of (3.22) in (3.13) leads to

$$K^2 = \omega^2+\epsilon\omega+\tfrac{1}{6}\epsilon^2+O(\epsilon^3). \quad (3.23)$$

If $b/a < 2$, an application of the binomial theorem, and substitution from (3.19), are then found to yield

$$K = \frac{\lambda}{2}(a+b)-\frac{\lambda}{24a}(b-a)^2+\dots. \quad (3.24)$$

In the customary method of evaluation of estuarial tides, in which the solution is dependent solely upon a curvilinear co-ordinate along the middle of the canal, the corresponding value of K is $\frac{1}{2}\lambda(a+b)$. With values of a and b for which (typically) $b/a \simeq 1{\cdot}5$, the additional term in (3.24) amounts to less than about 1 % of the elementary approximation.

The solutions of (3.16) are more easily obtained by assuming that μ is in some sense large. The analysis is effected by using the asymptotic development of a Bessel function of numerically large order (Watson 1958). From this, we find that

$$\phi_{i\mu}(\lambda r) \propto \cos\left(\mu\log\frac{a}{r}\right)+\frac{\lambda^2(r^2-a^2)}{4\mu}\sin\left(\mu\log\frac{a}{r}\right)+O\left(\frac{1}{\mu^2}\right).\dagger \quad (3.25)$$

The use of (3.25) in (3.16) then reveals that μ satisfies

$$\tan\left(\mu\log\frac{a}{b}\right) = \frac{\lambda^2(b^2-a^2)}{4\mu}+O\left(\frac{1}{\mu^2}\right), \quad (3.26)$$

which yields

$$\mu\log\frac{b}{a} = s\pi-\tan^{-1}\left\{\frac{\lambda^2(b^2-a^2)}{4\mu}+O\left(\frac{1}{\mu^2}\right)\right\} \quad (s = 1, 2, \dots). \quad (3.27)$$

Upon neglect of terms $O(1/\mu^2)$, this reduces to

$$\mu\log\frac{b}{a} = s\pi-\frac{\lambda^2(b^2-a^2)}{4\mu}, \quad (3.28)$$

from which we obtain the approximate solutions

$$\mu = \mu_s = \frac{s\pi}{\log(b/a)}-\frac{\epsilon^2}{4s\pi}\left(\frac{b+a}{b-a}\right). \quad (3.29)$$

The first term in (3.29) agrees with Cochran (1964). For tidal conditions in canals of realistic estuarial dimensions, the first term completely dominates the second and will accordingly furnish an excellent approximation to μ.

† In the subsequent numerical computations, the factor of proportionality is taken as

$$\frac{i}{2\pi\lambda a}\exp(\mu\pi).$$

4. Determination of the constants

The arbitrary constants in the solutions of §3 will here be determined by application of the juncture conditions (2.9) and (2.10).

Upon use of the formal solutions (3.6) and (3.17), it is found that (2.9) leads to the requirements

$$A+\sum_{n=1}^{\infty} C_n \cos\left\{\frac{n\pi(r-a)}{b-a}\right\} = \phi_K(\lambda r)\,(\alpha\cos\tfrac{1}{2}K\pi+\beta\sin\tfrac{1}{2}K\pi) + \sum_{s=1}^{\infty}\phi_{i\mu_s}(\lambda r)\,[\gamma_s\exp(\mu_s\tfrac{1}{2}\pi)+\delta_s\exp(-\mu_s\tfrac{1}{2}\pi)] \tag{4.1}$$

and

$$\lambda B-\sum_{n=1}^{\infty} p_n C_n \cos\left\{\frac{n\pi(r-a)}{b-a}\right\} = \frac{K}{r}(\alpha\sin\tfrac{1}{2}K\pi-\beta\cos\tfrac{1}{2}K\pi)\,\phi_K(\lambda r) - \frac{1}{r}\sum_{s=1}^{\infty}\mu_s\phi_{i\mu_s}(\lambda r)\,[\gamma_s\exp(\mu_s\tfrac{1}{2}\pi)-\delta_s\exp(-\mu_s\tfrac{1}{2}\pi)], \tag{4.2}$$

since, at $\theta=\frac{1}{2}\pi$, y and r have the same significance. These relations must be satisfied for all values of r in the interval (a,b). It is therefore necessary to develop the right-hand sides of (4.1) and (4.2) as suitable Fourier series and then, in the resulting expressions, to compare the coefficients of like orthogonal functions. With regard to (4.1), this procedure is found to yield the relations

$$(b-a)A = E(\alpha,\beta)\int_a^b \chi_K(r,0)\,dr+\sum_{s=1}^{\infty}[\gamma_s\exp(\mu_s\tfrac{1}{2}\pi)+\delta_s\exp(-\mu_s\tfrac{1}{2}\pi)]\int_a^b \chi_s(r,0)\,dr, \tag{4.3}$$

$$\tfrac{1}{2}(b-a)\,C_n = E(\alpha,\beta)\int_a^b \chi_K(r,n)\,dr+\sum_{s=1}^{\infty}[\gamma_s\exp(\mu_s\tfrac{1}{2}\pi)+\delta_s\exp(-\mu_s\tfrac{1}{2}\pi)] \times\int_a^b \chi_s(r,n)\,dr \quad (n=1,2,\ldots), \tag{4.4}$$

where

$$E(\alpha,\beta)=\alpha\cos\tfrac{1}{2}K\pi+\beta\sin\tfrac{1}{2}K\pi \tag{4.5}$$

and

$$\left.\begin{aligned}\chi_K(r,n) &= \phi_K(\lambda r)\cos\left\{\frac{n\pi(r-a)}{b-a}\right\},\\ \chi_s(r,n) &= \phi_{i\mu_s}(\lambda r)\cos\left\{\frac{n\pi(r-a)}{b-a}\right\}.\end{aligned}\right\} \tag{4.6}$$

Likewise, (4.2) leads to

$$\lambda(b-a)B = KF(\alpha,\beta)\int_a^b \frac{\chi_K(r,0)}{r}\,dr - \sum_{s=1}^{\infty}\mu_s[\gamma_s\exp(\mu_s\tfrac{1}{2}\pi)-\delta_s\exp(-\mu_s\tfrac{1}{2}\pi)]\int_a^b\frac{\chi_s(r,0)}{r}\,dr \tag{4.7}$$

and

$$-\tfrac{1}{2}(b-a)\,p_n C_n = KF(\alpha,\beta)\int_a^b\frac{\chi_K(r,n)}{r}\,dr - \sum_{s=1}^{\infty}\mu_s[\gamma_s\exp(\mu_s\tfrac{1}{2}\pi)-\delta_s\exp(-\mu_s\tfrac{1}{2}\pi)]\int_a^b\frac{\chi_s(r,n)}{r}\,dr \quad (n=1,2,\ldots), \tag{4.8}$$

where

$$F(\alpha,\beta)=\alpha\sin\tfrac{1}{2}K\pi-\beta\cos\tfrac{1}{2}K\pi. \tag{4.9}$$

Upon elimination of C_n between (4.4) and (4.8), and the subsequent elimination of $E(\alpha,\beta)$ and $F(\alpha,\beta)$ from the resultant by use of (4.3) and (4.7), we obtain an infinite sequence of relations between γ_s, δ_s, A and B. Similarly, by application of (2.10), we find that

$$(b-a)G = \{E(\alpha,\beta)\cos K\theta_1 - F(\alpha,\beta)\sin K\theta_1\}\int_a^b \chi_K(r,0)\,dr + \sum_{s=1}^{\infty}[\gamma_s \exp\{\mu_s(\theta_1+\tfrac{1}{2}\pi)\} + \delta_s \exp\{-\mu_s(\theta_1+\tfrac{1}{2}\pi)\}]\int_a^b \chi_s(r,0)\,dr, \quad (4.10)$$

$$\tfrac{1}{2}(b-a)D_n = \{E(\alpha,\beta)\cos K\theta_1 - F(\alpha,\beta)\sin K\theta_1\}\int_a^b \chi_K(r,n)\,dr + \sum_{s=1}^{\infty}[\gamma_s \exp\{\mu_s(\theta_1+\tfrac{1}{2}\pi)\} + \delta_s \exp\{-\mu_s(\theta_1+\tfrac{1}{2}\pi)\}]\int_a^b \chi_s(r,n)\,dr \quad (n = 1,2,\ldots), \quad (4.11)$$

$$\lambda(b-a)H = K\{E(\alpha,\beta)\cos K\theta_1 + F(\alpha,\beta)\sin K\theta_1\}\int_a^b \frac{\chi_K(r,0)}{r}\,dr - \sum_{s=1}^{\infty}\mu_s[\gamma_s \exp\{\mu_s(\theta_1+\tfrac{1}{2}\pi)\} - \delta_s \exp\{-\mu_s(\theta_1+\tfrac{1}{2}\pi)\}]\int_a^b \frac{\chi_s(r,0)}{r}\,dr \quad (4.12)$$

and

$$\tfrac{1}{2}(b-a)p_n D_n = K\{E(\alpha,\beta)\cos K\theta_1 + F(\alpha,\beta)\sin K\theta_1\}\int_a^b \frac{\chi_K(r,n)}{r}\,dr - \sum_{s=1}^{\infty}\mu_s[\gamma_s \exp\{\mu_s(\theta_1+\tfrac{1}{2}\pi)\} - \delta_s \exp\{-\mu_s(\theta_1+\tfrac{1}{2}\pi)\}]\int_a^b \frac{\chi_s(r,n)}{r}\,dr \quad (n = 1,2,\ldots). \quad (4.13)$$

Upon elimination of D_n between (4.11) and (4.13) and substitution for $E(\alpha,\beta)$ and $F(\alpha,\beta)$ from (4.3) and (4.7), we obtain a second infinite sequence of relations between γ_s, δ_s, A and B. For prescribed values of A and B, we may therefore express γ_s and δ_s ($s = 1,2,\ldots$) as the ratio between certain determinants of infinite order. In a practical evaluation, however, it is necessary to truncate the infinite series at an appropriate stage, thus obtaining, after a finite number of operations, an approximation to the various coefficients. This procedure presupposes, of course, the convergence of all the infinite processes. We may therefore regard suitable approximations to γ_s and δ_s as having been determined for $1 \leqslant s \leqslant m$, say.

Appropriate representations for G and H may now be obtained by calculating $E(\alpha,\beta)$ and $F(\alpha,\beta)$ from (4.3) and (4.7) and then using (4.10) and (4.12). The complete analytical expressions are lengthy, but are without attendant difficulties as regards numerical computation.

5. Numerical evaluation

In the present paper, the various infinite series representing the transverse oscillations will be truncated after one term ($m = 1$). The accuracy of this reduction will be considered later. Furthermore, we choose the spatial phase of the waves so that $B = 0$. With these simplifications, the problem reduces (essentially) to the determination of the constants γ_1 and δ_1. These are now solutions of a relatively simple pair of simultaneous equations, the coefficients in which involve the integrals detailed in §4. The values of these integrals may be obtained by numerical quadrature or, if ϵ be sufficiently small, by power series expansion in terms of ϵ. In the present work, both methods (when applicable) have been used, but the detail is not given here.

On the grounds of energy conservation, it is evident that the amplitudes of the standing waves in regions (2) and (3) must be identical at great distances from the bend. This requirement necessitates the relation

$$A^2 = G^2 + H^2, \tag{5.1}$$

	C_1/A	α/A	β/A	γ_1/Ai
(i)	$-1{\cdot}92\times10^{-5}$	$0{\cdot}376$	$0{\cdot}011$	$-8{\cdot}41\times10^{-24}$
(ii)	$-0{\cdot}207$	$-3{\cdot}91$	$2{\cdot}05$	$-3{\cdot}70\times10^{-14}$
	δ_1/Ai	D_1/A	G/A	H/A
(i)	$-6{\cdot}17\times10^{-12}$	$-2{\cdot}36\times10^{-5}$	$0{\cdot}999$	$2{\cdot}61\times10^{-2}$
(ii)	$-3{\cdot}08\times10^{-4}$	$-0{\cdot}17$	$0{\cdot}881$	$-0{\cdot}465$

TABLE 1. Numerical values of coefficients in solution: (i) $\sigma = 1{\cdot}45\times10^{-4}\ \mathrm{sec}^{-1}$; (ii) $\sigma = 3\times10^{-2}\ \mathrm{sec}^{-1}$

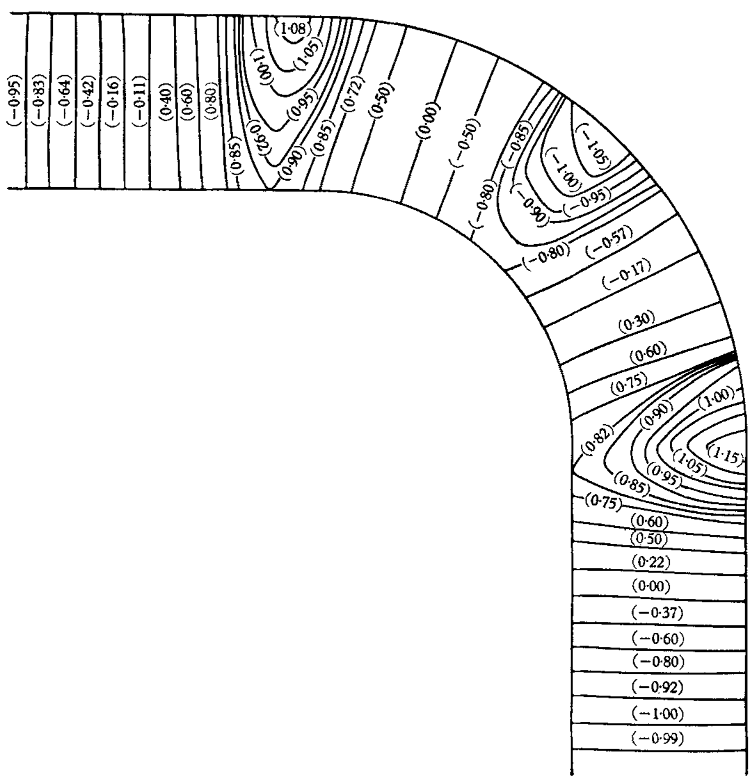

FIGURE 2. Contours of equal values of Z/A in case (ii).

the approximate fulfilment of which will be taken to justify the before-mentioned truncation processes.

In both the numerical examples considered here, we take $a = 1000$ m, $b = 1700$ m, $h = 12$ m and $\theta_1 = 90°$. In case (i) we take $\sigma = 1{\cdot}45 \times 10^{-4}\,\text{sec}^{-1}$ whilst in case (ii) $\sigma = 3 \times 10^{-2}\,\text{sec}^{-1}$. The values of the various coefficients in (3.6), (3.7) and (3.17) are then given in table 1. In both cases, we see that the results are consistent with (5.1). In case (i) the relatively small values of C_1 and D_1 indicate that the tidal dynamics may effectively be deduced from the rectilinear models referred to in §1.

The contours of equal values of Z/A within the canal, corresponding to case (ii), are given in figure 2. From this, it is evident that the maximum wave amplitudes are attained at the outside of the bend. These amount to about a 20% increase over the amplitude of the primary oscillation.

Finally, we observe that these dynamical effects are essentially confined to within a distance from the bend less than the width of the canal. Accordingly, it is permissible to insert a barrier across the canal at a sequence of positions in region (3). The model is then applicable to the case of a tsunami-type oscillation in a curved canal which is closed at its landward end.

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